Week 10 (disc 105)

Wednesday, March 31, 2021 9:02 AM

4.9 Problems

Problem 1. *Use Composite Simpson's rule and the given value of n to approximate the following improper integrals:*

1. $\int_0^1 x^{-1/4}\sin(x)dx$, $n = 4$
2. $\int_0^1 \frac{e^{2x}}{x^{2/5}}dx$, $n = 6$

Problem 2. *Use the transformation $t = x^{-1}$ and the composite Simpson's rule for $n = 4$ to compute:*

$$\int_1^\infty \frac{1}{x^2+9}dx$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$e^{ix} = \cos(x) + i\sin(x)$$

1) $f(x) = \sin(x)$ $\qquad P_4 := x - \frac{x^3}{3!}$

$$\int_0^1 \frac{\sin(x)}{x^{1/4}}dx = \underbrace{\int_0^1 \frac{\overset{G(x)}{\sin(x) - P_4(x)}}{x^{1/4}}dx}_{A} + \underbrace{\int_0^1 \frac{P_4(x)}{x^{1/4}}dx}_{B}$$

$$B = \int_0^1 x^{3/4} - \frac{x^{12/4}}{6x^{1/4}}dx = \dots = \frac{166}{315}$$

$$A = \frac{.25}{3}\left[G(0) + 4G(.25) + 2G(.5) + 4G(.75) + G(1)\right] \cong .00132$$

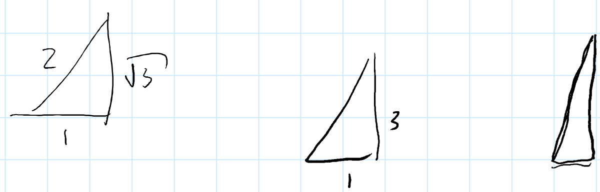
2. $\displaystyle\int_1^\infty \frac{1}{x^2+9}dx$

$t = 1/x \qquad x = 1/t \qquad dx = \frac{-1}{t^2}dt$

$x = 1 \Rightarrow t = 1$

$x = \infty \Rightarrow t = 0$

$$\int_1^0 \frac{1}{t^{-2}+9}\frac{-1}{t^2}dt = \int_0^1 \frac{1}{1+9t^2}dt = \frac{1}{3}\arctan(3t)\Big|_0^1 = \frac{1}{3}\arctan(3)$$



Problem 3. *Use Theorem 5.4 to show that the following initial-value problems have a unique solution, and find the solution:*

1. $y' = y\cos(t)$, $0 \le t \le 1$, $y(0) = 1$
2. $y' = -\frac{2}{t}y + t^2e^t$, $1 \le t \le 2$, $y(1) = \sqrt{2}e$

Problem 4. *Show that the given equation implicitly defines a solution. Approximate $y(2)$ using Newton's method:*

$$y' = -\frac{y^3+y}{(3y^2+1)t}$$

for $1 \le t \le 2$, $y(1) = 1$. For the equation: $y^3y + yt = 2$

$y' = y\cos(t)$

$\frac{dy}{y} = \cos(t)\,dt$

$\log|y| = \sin(t) + c$

$|y| = y(0)\,e^{\sin(t)}$

$|y(t)| = y(0)\,e^{\sin(t)}$

$\boxed{y(t) = e^{\sin(t)}}$

3) 1.) $y' = f(t,y) \qquad \underline{f(t,y) = y\cos(t)}$

NTS $|f(t,y_1) - f(t,y_2)| \le L\,|y_1 - y_2| \quad \forall\ y_1 \ne y_2$

$$\left|\frac{f(t,y_1) - f(t,y_2)}{y_1 - y_2}\right| = \left|\partial_y f(t,\xi)\right|$$

L

$\Rightarrow \; |f(t,y_1) - f(t,y_2)| \le |y_1 - y_2| \cdot 1$ L

$$y' = -\frac{2}{t} y + t^2 e^t = f(t,y) \qquad 1 \le t \le 2$$

$$\left|\partial_y f(t,y)\right| = \left|-\frac{2}{t}\right| \le \frac{2}{t} \le 2$$

$1 \le t \le 2$

$1 \ge \frac{1}{t} \ge \frac{1}{2}$

$2 \ge \frac{2}{t} \ge 1$

$\boxed{\text{integrating factor}}$

$$y' + \frac{2}{t} y = t^2 e^t$$

multiply by t^2

$$\underbrace{y' t^2 + 2yt}_{\frac{d}{dt}(y t^2)} = t^4 e^t$$

$$\frac{d}{dt}(y t^2) = t^4 e^t$$

$$y t^2 = \underbrace{\int_1^t s^4 e^s \, ds}\; + \; C$$

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if y satisfies $y^3 t + yt = 2$ $**$

$\Rightarrow$ y' satisfies

$\frac{d}{dt}(**)$ $\quad 3y^2 y' t + y^3 + y' t + y = 0$

$$y' = \frac{-y - y^3}{3y^2 t + t}$$